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A method for determining anomalous gauge boson couplings from e^+e^- experiments

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ABSTRACT

We present a model-independent method for determining anomalous gauge boson couplings from ongoing and future $e^+e^- \to W^+W^-$ experiments. First we generalize an already existing method, which relies on the study of four observables constructed through appropriate projections of the unpolarized differential cross-section. In particular, we retain both linear and quadratic terms in the unknown couplings, and compute contributions to these observables originating from anomalous couplings which do not separately conserve the discrete C, P, and T symmetries. Second, we combine the above set of observables with three additional ones, which can be experimentally obtained from the total cross-sections for polarized final state W bosons. The resulting set of seven observables may provide useful information for constraining, and in some cases for fully determining, various of the possible anomalous gauge boson couplings.

1 Introduction

Anomalous gauge boson couplings [1,2] have attracted significant attention in recent years, and their direct study through the process $e^+e^- \rightarrow W^+W^-$ has been one of the main objectives of the CERN Large Electron Positron collider LEP2 [3,4,5,6,7,8]. In addition, the trilinear gauge self-couplings have also been probed through direct $W\gamma$ and WZ production at the Tevatron [9,10]. The study of such couplings is expected to continue at the CERN Large Hadron Collider (LHC), as well as the Next Linear Collider (NLC) [11].

Recently a model-independent method has been proposed for extracting values or bounds for the anomalous gauge boson couplings from $e^+e^- \rightarrow W^+W^-$ experiments [12]. The basic idea is to study projections of the differential cross-section which arise when the latter is convoluted with a set of appropriately constructed polynomials in $\cos\theta$, where θ is the center-of-mass scattering angle. This construction leads to a set of four novel observables, which are related to the anomalous couplings by means of simple algebraic equations. The experimental determination of these observables can in turn be used in order to impose bounds simultaneously on all anomalous couplings, without having to resort to model-dependent relations among them, or invoke any further simplifying assumptions. This method has also been generalized to the case of hadron colliders [13], and its compatibility with the inclusion of structure function effects has been established. In what follows we will refer to this method as the "Projective Method" (PM).

The PM as presented in [12] only includes terms linear in the unknown anomalous couplings (form-factors) which are individually invariant under the discrete C, P, and T symmetries. However, the inclusion of the quadratic terms as well as the C, P, and T non-invariant couplings is necessary for a complete experimental analysis. In addition, the observables constructed by means of the PM are only four, whereas the unknown form-factors, even with the simplifications mentioned above, are six; therefore, one is not able to extract experimental information for all anomalous couplings, but only for a few of them. In addition, the fact that the observables constructed by this method are rather strongly correlated further reduces the predictive power of the PM.

The purpose of this paper is two-fold:

(i) The contribution of *all* anomalous couplings is computed, and terms linear and quadratic are retained. This not only augments the PM, but as we will see later, results in the additional advantage of reducing the correlation among the four original PM observables.

(ii) The aforementioned four observables of the PM are combined with three additional observables, which can be extracted, at least in principle, from measurements of polarized total cross-sections. Specifically, the observables correspond to the total cross-section for having two transverse, two longitudinal, and one transverse and one longitudinal W bosons in the final state. These quantities have already been studied in the literature [14], and are usually denoted by σ_{TT}, σ_{LL}, and σ_{TL}, respectively. In fact, it is expected that experimental values for the aforementioned observables can be extracted from the available LEP2 experimental data [15].

The inclusion of σ_{TT}, σ_{LL}, and σ_{TL} to the original PM observables gives rise to a set of seven observables, thus increasing the predictive power of the method. For practical purposes in this paper we present the case where the aforementioned three cross-sections are calculated keeping linear and quadratic parts of the C, P, and T invariant couplings only. To the best of our knowledge the explicit closed form of these cross-sections in terms of the anomalous couplings is presented here for the first time.

The outline of the paper is as follows: In section 2 we present the complete expressions for the PM observables, keeping all terms. In section 3 we compute the closed expressions for the polarized cross-sections, keeping quadratic correction but, assuming the presence of C, and P invariant couplings only. In section 4 we focus on the system of equations obtained when the results of the previous two sections are combined; at this stage the linear and quadratic terms of the C and P invariant couplings are retained, giving rise to seven equations for six unknown couplings. We discuss various issues, and carry out an elementary analysis of the correlations among some of these observables. Finally, in section 5 we present our conclusions.

2 The complete expressions for the σ_i observables

In this section we extend the analysis presented in [12] by including the linear *and* quadratic contributions of *all* possible anomalous couplings. We consider the process $e^-(k_1,\sigma_1)e^+(k_2,\sigma_2) \to W^-(p_1,\lambda_1)W^+(p_2,\lambda_2)$, shown in Fig. 1 The electrons are assumed to be massless, σ_i label the spins of the initial electron and positron, i.e. $\sigma_1 = -\sigma_2 = \sigma/2,\ \sigma = \pm 1$, whereas the λ_i label the the polarizations of the produced W bosons, with $\lambda_i = 0, \pm 1$.

The relevant kinematical variables in the center-of-mass frame are

$$\begin{aligned} s = &\ (k_1+k_2)^2 = \ (p_1+p_2)^2 \ , \\ t = &\ (k_1-p_1)^2 = \ (p_2-k_2)^2 = -\frac{s}{4}(1+\beta^2-2\beta\cos\theta) = -\frac{s\beta}{2}(z-x) \ , \end{aligned} \tag{2.1}$$

where

$$\beta = \sqrt{1-\frac{4M_W^2}{s}} \ , \tag{2.2}$$

is the velocity of the W bosons, $x \equiv \cos\theta$, where θ is the angle between the incoming electron and the outgoing W^- in the center of mass frame, and

$$z = \frac{1+\beta^2}{2\beta} \ . \tag{2.3}$$

We now proceed to compute the unpolarized differential cross-section $(d\sigma/dx)$ corresponding to this process, i.e. we average over the initial spins and sum over the final polarizations. We have

$$\frac{d\sigma}{dx} = \left(\frac{1}{2s}\right)\left(\frac{\beta}{16\pi}\right)\frac{g^4}{4}\sum_{\sigma,\lambda_1,\lambda_2}|\mathcal{M}^\sigma(\lambda_1,\lambda_2)|^2 \ . \tag{2.4}$$

The first fraction is the flux factor, the second is a phase space factor, and the factor of 1/4 is due to the averaging over the initial helicities. All conventions are identical to those of [12] except that we have now pulled out the overall coupling constant factor and have denoted the remaining sum of amplitudes by $\mathcal{M}$.

The VW^+W^- vertex $\Gamma^V_{\mu\alpha\beta}$ $(V=\gamma,\ Z)$ we use has the form

$$\Gamma^V_{\mu\alpha\beta} = \Gamma^0_{\mu\alpha\beta} + \delta\Gamma^V_{\mu\alpha\beta} \ , \tag{2.5}$$

where

$$\Gamma^0_{\mu\alpha\beta}(q,-p_1,-p_2) = (p_2-p_1)_\mu g_{\alpha\beta} + 2(q_\beta g_{\mu\alpha} - q_\alpha g_{\beta\mu}) \tag{2.6}$$

is the canonical Standard Model (SM) three-boson vertex at tree-level, assuming that the two W-bosons are on-shell, and thus dropping terms proportional to $p_{1\alpha}$ and $p_{2\beta}$. The term $\delta\Gamma^V_{\mu\alpha\beta}$ contains all possible deviations from the SM canonical form, compatible with Lorentz invariance, i.e.

$$\begin{aligned}
\delta\Gamma^V_{\mu\alpha\beta}(q,-p_1,-p_2) \;=\;& f_1^V(p_2-p_1)^\mu g^{\alpha\beta} - \frac{f_2^V}{2M_W^2} q^\alpha q^\beta (p_2-p_1)^\mu \\
&+2f_3^V(q^\beta g^{\mu\alpha} - q^\alpha g^{\beta\mu}) \\
&+if_4^V(q^\beta g^{\mu\alpha} + q^\alpha g^{\beta\mu}) + if_5^V \epsilon^{\mu\alpha\beta\rho}(p_2-p_1)_\rho \\
&+f_6^V \epsilon^{\mu\alpha\beta\rho} q_\rho + \frac{f_7^V}{M_W^2}(p_2-p_1)^\mu \epsilon^{\alpha\beta\rho\sigma} q_\rho (p_2-p_1)_\sigma \;. \qquad (2.7)
\end{aligned}$$

The deviation form-factors f_i^V are all zero in SM. In what follows they will also be referred to as trilinear couplings or anomalous couplings. We assume all anomalous couplings to be real.

The calculation is straightforward but lengthy; it is important to emphasize that the inclusion of the additional terms in the vertex, namely those that are not separately C and P invariant $(f_4^V, f_5^V, f_6^V, f_7^V)$ does *not* change the functional dependence of the differential cross-section on the center-of-mass angle θ, which was established in [12]. Thus, the expression for $(d\sigma_{an}/dx)$, the part of the differential cross-section which contains the anomalous couplings, assumes again the form

$$(z-x)\frac{d\sigma_{an}}{dx} = \frac{g^4}{64\pi}\frac{\beta}{s}\sum_{i=1}^{4}\sigma_i(s)P_i(s,x) \qquad (2.8)$$

with $P_i(s,x)$ the same polynomials in x first obtained in [12] namely :

$$\begin{aligned}
P_1(x) &= z-x \;, \\
P_2(x) &= (z-x)(1-x^2) \;, \\
P_3(x) &= 1-x^2 \;, \\
P_4(x) &= 1-\beta x \;. \qquad (2.9)
\end{aligned}$$

In arriving at this result the following algebraic identities

$$\begin{aligned}
x(z-x) &= -\frac{1}{\beta}P_1 + P_3 + \frac{1}{2\eta\beta^2}P_4 \\
x-\beta &= -2P_1 + \frac{1}{\beta}P_4 \qquad (2.10)
\end{aligned}$$

may be found useful.

Notice that the explicit closed expressions of the coefficients σ_i have changed with respect to those reported in [12], since both the *linear* as well as the *quadratic* dependence on *all* couplings has now been included. Using the following uniform short-hand notation

$$c_{1,\ldots,7} \equiv f^\gamma_{1,\ldots,7} \;, \qquad c_{8,\ldots,14} \equiv f^Z_{1,\ldots,7} \;, \qquad (2.11)$$

we have that the σ_i, where $i = 1, \ldots 4$, can be written as

$$\sigma_i = \sum_k L^i_k \, c_k + \sum_k \sum_{\ell \geq k} Q^i_{[k][\ell]} \, c_k c_\ell \;. \qquad (2.12)$$

Defining the following abbreviations

$$\eta \equiv \frac{s}{4M_W^2}, \quad u \equiv \frac{s}{s-M_Z^2}, \quad y \equiv \frac{u}{c_w^2}, \quad r \equiv v^2 + a^2 \ , \tag{2.13}$$

the explicit forms of the coefficients L_k^i and $Q^i_{[k][\ell]}$ of the linear and quadratic terms respectively in Eq.(2.12) are given below :

$$\begin{array}{ll} L_3^1 = -8s_w^2[4s_w^2 + v(4c_w^2-1)y] \ , & L_5^1 = 4s_w^2[a(4c_w^2-1)y-1] \ , \\ L_{10}^1 = -8u[4vs_w^2 + r(4c_w^2-1)y] \ , & L_{12}^1 = -4(v+a)u + 8au[va(4c_w^2-1)y + 2s_w^2] \ . \end{array} \tag{2.14}$$

$$\begin{array}{llll} Q^1_{[3][3]} & = 16s_w^4\eta\beta^2 \ , & Q^1_{[3][10]} & = 32vs_w^2\eta\beta^2 u \ , \\ Q^1_{[3][12]} & = -16as_w^2\eta\beta^2 u \ , & Q^1_{[4][4]} & = 4s_w^4\eta\beta^2 \ , \\ Q^1_{[4][11]} & = 8vs_w^2\eta\beta^2 u \ , & Q^1_{[4][13]} & = -8as_w^2\eta u \ , \\ Q^1_{[5][5]} & = 4s_w^4\eta\beta^4 \ , & Q^1_{[5][10]} & = -16as_w^2\eta\beta^2 u \ , \\ Q^1_{[5][12]} & = 8vs_w^2\eta\beta^4 u \ , & Q^1_{[6][6]} & = 4s_w^4\eta \ , \\ Q^1_{[6][11]} & = -8as_w^2\eta u \ , & Q^1_{[6][13]} & = 8vs_w^2\eta u \ , \\ Q^1_{[10][10]} & = 16\eta\beta^2 ru^2 \ , & Q^1_{[10][12]} & = -32va\eta\beta^2 u^2 \ , \\ Q^1_{[11][11]} & = 4\eta\beta^2 ru^2 \ , & Q^1_{[11][13]} & = -16va\eta u^2 \ , \\ Q^1_{[12][12]} & = 4\eta\beta^4 ru^2 \ , & Q^1_{[13][13]} & = 4\eta ru^2 \ . \end{array} \tag{2.15}$$

$$\begin{array}{ll} L_1^2 = s_w^2\beta^2[2(3-2\eta)(vu+s_w^2) - (1+2\eta)vy] \ , & L_2^2 = -2\beta^2\eta s_w^2[2(1+\eta)(vu+s_w^2) - \eta yv] \ , \\ L_3^2 = -4\beta^2\eta s_w^2[2(vu+s_w^2) - yv] \ , & L_8^2 = \beta^2 u\,[2(3-2\eta)(ru+vs_w^2) - (1+2\eta)yr] \ , \\ L_9^2 = -2\beta^2\eta u[2(1+\eta)(ru+vs_w^2) - \eta yr] \ , & L_{10}^2 = -4\beta^2\eta u[2(ru+vs_w^2) - yr] \ , \end{array} \tag{2.16}$$

$$\begin{array}{ll} Q^2_{[1][1]} = s_w^4\eta^2\beta^2(3-2\beta^2+3\beta^4) \ , & Q^2_{[1][2]} = -4s_w^4\eta^3\beta^4(1+\beta^2) \ , \\ Q^2_{[1][3]} = -8s_w^4\eta^2\beta^2(1+\beta^2) \ , & Q^2_{[1][8]} = 2vs_w^2\eta^2\beta^2(3-2\beta^2+3\beta^4)u \ , \\ Q^2_{[1][9]} = -4vs_w^2\eta^3\beta^4(1+\beta^2)u \ , & Q^2_{[1][10]} = -8vs_w^2\eta^2\beta^2(1+\beta^2)u \ , \\ Q^2_{[2][2]} = 4s_w^4\eta^4\beta^6 \ , & Q^2_{[2][3]} = 16s_w^4\eta^3\beta^4 \ , \\ Q^2_{[2][8]} = -4vs_w^2\eta^3\beta^4(1+\beta^2)u \ , & Q^2_{[2][9]} = 8vs_w^2\eta^4\beta^6 u \ , \\ Q^2_{[2][10]} = 16vs_w^2\eta^3\beta^4 u \ , & Q^2_{[3][3]} = 8s_w^4\eta^2\beta^2(1+\beta^2) \ , \\ Q^2_{[3][8]} = -8vs_w^2\eta^2\beta^2(1+\beta^2)u \ , & Q^2_{[3][9]} = 16vs_w^2\eta^3\beta^4 u \ , \\ Q^2_{[3][10]} = 16vs_w^2\eta^2\beta^2(1+\beta^2)u \ , & Q^2_{[4][4]} = -2s_w^4\eta\beta^2 \ , \\ Q^2_{[4][11]} = -4vs_w^2\eta\beta^2 u \ , & Q^2_{[5][5]} = -2s_w^4\eta\beta^4 \ , \\ Q^2_{[5][12]} = -4vs_w^2\eta\beta^4 u \ , & Q^2_{[6][6]} = -2s_w^4\eta\beta^2 \ , \\ Q^2_{[6][7]} = 16s_w^4\eta\beta^2 \ , & Q^2_{[6][13]} = -4vs_w^2\eta\beta^2 u \ , \\ Q^2_{[6][14]} = 16vs_w^2\eta\beta^2 u \ , & \\ Q^2_{[7][7]} = 32s_w^4\eta^2\beta^4 \ , & Q^2_{[7][13]} = 16vs_w^2\eta\beta^2 u \ , \\ Q^2_{[7][14]} = 64vs_w^2\eta^2\beta^4 u \ , & Q^2_{[8][8]} = \eta^2\beta^2 r(3-2\beta^2+3\beta^4)u^2 \ , \\ Q^2_{[8][9]} = -4\eta^3\beta^4 r(1+\beta^2)u^2 \ , & Q^2_{[8][10]} = -8\eta^2\beta^2 r(1+\beta^2)u^2 \ , \\ Q^2_{[9][9]} = 4\eta^4\beta^6 ru^2 \ , & Q^2_{[9][10]} = 16\eta^3\beta^4 ru^2 \ , \\ Q^2_{[10][10]} = 8\eta^2\beta^2 r(1+\beta^2)u^2 \ , & Q^2_{[11][11]} = -2\eta\beta^2 ru^2 \ , \\ Q^2_{[12][12]} = -2\eta\beta^4 ru^2 \ , & Q^2_{[13][13]} = -2\eta\beta^2 ru^2 \ , \\ Q^2_{[13][14]} = 16\eta\beta^2 ru^2 \ , & Q^2_{[14][14]} = 32\eta^2\beta^4 ru^2 \ . \end{array} \tag{2.17}$$

$$\begin{array}{ll} L_1^3 = -s_w^2\beta \ , & L_2^3 = s_w^2\eta\beta \ , \\ L_5^3 = s_w^2\beta[1-4a(4c_w^2-1)y] \ , & L_8^3 = -(v+a)\beta u \ , \\ L_9^3 = (v+a)\eta\beta u \ , & L_{12}^3 = \beta u[(v+a) - 8va(4c_w^2-1)y - 16as_w^2] \ . \end{array} \tag{2.18}$$

$$\begin{array}{ll} Q^3_{[3][12]} = 16as_w^2\eta\beta^3 u \ , & Q^3_{[4][13]} = 8as_w^2\eta\beta u \ , \\ Q^3_{[5][10]} = 16as_w^2\eta\beta^3 u \ , & Q^3_{[6][11]} = 8as_w^2\beta\eta u \ , \\ Q^3_{[10][12]} = 32va\eta\beta^3 u^2 \ , & Q^3_{[11][13]} = 16va\eta\beta u^2 \ . \end{array} \tag{2.19}$$

$$\begin{array}{ll} L_3^4 = 4s_w^2\beta^{-1} \ , & L_5^4 = -4as_w^2(4c_w^2-1)(z-\beta)y + 2s_w^2\beta^{-1} \ , \\ L_{10}^4 = 4(v+a)u\beta^{-1} \ , & L_{12}^4 = -8(z-\beta)[va(4c_w^2-1)y + 2as_w^2]u + 2(v+a)u\beta^{-1} \ . \end{array} \tag{2.20}$$

$$\begin{array}{ll} Q^4_{[3][12]} = 8as_w^2\beta u \ , & Q^4_{[4][13]} = 4as_w^2\beta^{-1}u \ , \\ Q^4_{[5][10]} = 8as_w^2\beta u \ , & Q^4_{[6][11]} = 4as_w^2\beta^{-1}u \ , \\ Q^4_{[10][12]} = 16va\beta u^2 \ , & Q^4_{[11][13]} = 8va\beta^{-1}u^2 \ . \end{array} \tag{2.21}$$

As explained in [12] the four quantities σ_i constitute a set of observables; their experimental values may be obtained through an appropriate convolution of the experimentally measured unpolarized differential cross-section $d\sigma^{(exp)}/dx$ with a set of four polynomials, $\widetilde{P}_i(x)$, which are orthonormal to the $P_i(x)$, i.e. they satisfy

$$\int_{-1}^{1} \widetilde{P}_i(x,s) P_j(x,s) dx = \delta_{ij} \ . \tag{2.22}$$

Clearly the set $\widetilde{P}_i$ is not uniquely determined; in [12] we have reported the set with the lowest possible power in x, namely :

$$\begin{aligned} \widetilde{P}_1(x,s) &= \frac{\eta}{2}(3\beta + 15x - 15\beta x^2 - 35x^3) \ , \\ \widetilde{P}_2(x,s) &= \frac{35}{8}(-3x + 5x^3) \ , \\ \widetilde{P}_3(x,s) &= \frac{5}{8}(3 + 21zx - 9x^2 - 35zx^3) \ , \\ \widetilde{P}_4(x,s) &= \frac{\eta}{2}(-3 - 15zx + 15x^2 + 35zx^3) \ . \end{aligned} \tag{2.23}$$

In particular, the $\sigma_i^{(exp)}$ are given by

$$\sigma_i^{(exp)} = \left[\frac{64\pi s}{g^4\beta}\right] \int_{-1}^{1} dx (z-x) \left(\frac{d\sigma^{(exp)}}{dx} - \frac{d\sigma^{(0)}}{dx}\right) \widetilde{P}_i(x,s) \ , \tag{2.24}$$

where $d\sigma^{(0)}/dx$ is the SM expression for the differential cross-section in the absence of anomalous couplings [16]. Given the experimental measurement of the differential cross-section $d\sigma^{(exp)}/dx$ for

on shell Ws the four numbers σ_i can be extracted together with their related errors. Subsequently Eq.(2.12) can be viewed as a system of four quadratic equations with fourteen unknowns which although cannot be solved, it appears feasible that it could be fitted for all couplings simultaneously in a model independent way. In fact, using $U(1)$ electromagnetic gauge invariance the photonic couplings f_1^γ and f_2^γ are related by $f_1^\gamma = \eta f_2^\gamma$, thus reducing the total number of unknowns to thirteen .

3 Polarized cross-sections

In this section we will augment the previous set of observables, which were projected out of the unpolarized differential cross-section, with three additional observables obtained from measurements of polarized total cross-sections. As a first step we will only compute in this section the polarized cross-sections obtained using non standard couplings that separately respect C and P, i.e., we only retain the first three f_1^V, f_2^V, f_3^V. In order to calculate the polarized cross-sections we define the following basic matrix elements for the production of two Ws with definite helicity from polarized e^-e^+ beams. For massless electrons the helicity of the positron is opposite to the polarization of the electron: $\sigma_1 = -\sigma_2 = \sigma$. Three basic matrix elements are defined, one for each of the first three terms of the trilinear gauge vertex in Eq.(2.7), and a fourth one for the neutrino exchange t-channel graph (Fig.1c):

$$\begin{aligned}
\mathcal{M}_1^\sigma(\lambda_1,\lambda_2) &= [\bar{v}(k_2,-\sigma)\not{p}_2 P_\sigma u(k_1,\sigma)]\,(\epsilon_{\lambda_1}(p_1)\cdot\epsilon_{\lambda_2}(p_2)) \\
\mathcal{M}_2^\sigma(\lambda_1,\lambda_2) &= [\bar{v}(k_2,-\sigma)\not{p}_1 P_\sigma u(k_1,\sigma)]\,(p_1\cdot\epsilon_{\lambda_2}(p_2))\,(p_2\cdot\epsilon_{\lambda_1}(p_1))\,\frac{1}{2M_W^2} \\
\mathcal{M}_3^\sigma(\lambda_1,\lambda_2) &= \bar{v}(k_2,-\sigma)\,[\not{\epsilon}_{\lambda_1}(p_1)\,(p_1\cdot\epsilon_{\lambda_2}(p_2)) - \not{\epsilon}_{\lambda_2}(p_2)\,(p_2\cdot\epsilon_{\lambda_1}(p_1))]\,P_\sigma u(k_1,\sigma) \\
\mathcal{M}_4^\sigma(\lambda_1,\lambda_2) &= \bar{v}(k_2,-\sigma)\not{\epsilon}_{\lambda_2}(p_2)(\not{k}_1-\not{p}_1)\not{\epsilon}_{\lambda_1}(p_1)P_\sigma u(k_1,\sigma)
\end{aligned} \tag{3.1}$$

where the helicity projectors are given by

$$P_\pm = \frac{1\pm\gamma_5}{2}\;. \tag{3.2}$$

We now establish contact with the notation of the previous section and that of [12]. In terms of the basic matrix elements, defined above, the amplitudes corresponding to the three graphs of the W pair-production process in Fig. 1 are expressed as :

$$\begin{aligned}
\mathcal{M}_\gamma^\sigma(\lambda_1,\lambda_2) &= \frac{2s_w^2}{s}\,[(1+f_1^\gamma)\mathcal{M}_1^\sigma(\lambda_1,\lambda_2) + f_2^\gamma\mathcal{M}_2^\sigma(\lambda_1,\lambda_2) + (1+f_3^\gamma)\mathcal{M}_3^\sigma(\lambda_1,\lambda_2)] \\
\mathcal{M}_Z^\sigma(\lambda_1,\lambda_2) &= \frac{2g_\sigma}{s-M_Z^2}\left[(1+f_1^Z)\mathcal{M}_1^\sigma(\lambda_1,\lambda_2) + f_2^Z\mathcal{M}_2^\sigma(\lambda_1,\lambda_2) + (1+f_3^Z)\mathcal{M}_3^\sigma(\lambda_1,\lambda_2)\right] \\
\mathcal{M}_\nu^\sigma(\lambda_1,\lambda_2) &= -\frac{1}{2t}\mathcal{M}_4^\sigma(\lambda_1,\lambda_2)\delta_{\sigma-}
\end{aligned} \tag{3.3}$$

where an overall coupling constant factor of ig^2 has been pulled out, the left and right handed couplings of the electron with the Z boson are given by

$$g_+ = v - a\;,\quad g_- = v + a, \tag{3.4}$$

and the Kronecker δ ($\delta_{--} = \delta_{++} = 1$, $\delta_{-+} = \delta_{+-} = 0$) in the neutrino graph appears due to the fact that the W bosons couple only to left handed electrons.

The full amplitude can then be cast in the form:

$$\begin{aligned}
\mathcal{M}^\sigma(\lambda_1,\lambda_2) &= \mathcal{M}^\sigma_\gamma(\lambda_1,\lambda_2) + \mathcal{M}^\sigma_Z(\lambda_1,\lambda_2) + \mathcal{M}^\sigma_\nu(\lambda_1,\lambda_2) \\
&= \frac{1}{s}\sum_{i=1}^{4} F_i^\sigma \mathcal{M}_i^\sigma(\lambda_1,\lambda_2)
\end{aligned} \tag{3.5}$$

where

$$\begin{aligned}
F_i^\sigma &= 2s_w^2(1+f_i^\gamma) + 2g_\sigma u(1+f_i^Z), \qquad\qquad \text{for } i=1,3 \\
F_2^\sigma &= 2s_w^2 Q_e f_2^\gamma + 2g_\sigma u f_2^Z \ , \\
F_4^\sigma &= -\frac{s}{2t}\delta_{\sigma -} \ ,
\end{aligned} \tag{3.6}$$

are explicit functions of the anomalous couplings.

We then calculate the total cross-sections for the production of: (i) two transversely polarized Ws denoted by σ_T, (ii) of two longitudinally polarized Ws called σ_L and (iii) one transverse and one longitudinal W, denoted σ_M. We will present their explicit expressions in terms of arbitrary trilinear gauge couplings f_1^V, f_2^V, f_3^V. The relevant differential polarized cross-sections are defined by:

$$\begin{aligned}
\frac{d\sigma_T}{dx} &= \frac{1}{2s}\frac{\beta}{16\pi}\frac{g^4}{4}\sum_{\sigma,\lambda_1,\lambda_2=\pm} |\mathcal{M}(\sigma,\lambda_1,\lambda_2)|^2 \ , \\
\frac{d\sigma_M}{dx} &= \frac{1}{2s}\frac{\beta}{16\pi}\frac{g^4}{4}\sum_{\sigma,\lambda=\pm} \left[|\mathcal{M}(\sigma,\lambda,0)|^2 + |\mathcal{M}(\sigma,0,\lambda)|^2\right] \ , \\
\frac{d\sigma_L}{dx} &= \frac{1}{2s}\frac{\beta}{16\pi}\frac{g^4}{4}\sum_{\sigma=\pm} |\mathcal{M}(\sigma,0,0)|^2 \ .
\end{aligned} \tag{3.7}$$

These are calculated in a straightforward manner using the expressions of the basic matrix elements for the different polarization combinations. The non-vanishing amplitudes are explicitly given below:

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$$\begin{aligned}
\mathcal{M}_1(\sigma,\pm,\pm) &= -\frac{\beta s}{2}\sqrt{1-x^2} \ , \\
\mathcal{M}_4(\sigma,\pm,\pm) &= -\frac{\beta s}{2}(x-\beta)\sqrt{1-x^2} \ , \\
\mathcal{M}_4(\sigma,\pm,\mp) &= -\frac{s}{2}(x\mp\sigma)\sqrt{1-x^2} \ ,
\end{aligned} \tag{3.8}$$

<u>TL</u>

$$\begin{aligned}
\mathcal{M}_3(\sigma,\pm,0) = \mathcal{M}_3^\sigma(0,\mp) &= \sqrt{2\eta}\frac{\beta s}{2}(x\mp\sigma) \ , \\
\mathcal{M}_4(\sigma,\pm,0) = \mathcal{M}_4^\sigma(0,\mp) &= -\sqrt{2\eta}\frac{s}{4}[2(\beta-x)\mp\sigma/\eta)(x\mp\sigma) \ ,
\end{aligned} \tag{3.9}$$

LL

$$
\begin{aligned}
\mathcal{M}_1(\sigma,0,0) &= -\frac{\beta s(2\eta-1)}{2}\sqrt{1-x^2}\ , \\
\mathcal{M}_2(\sigma,0,0) &= \beta\eta s(\eta-1)\sqrt{1-x^2}\ , \\
\mathcal{M}_3(\sigma,0,0) &= 2\beta\eta s\sqrt{1-x^2}\ , \\
\mathcal{M}_4(\sigma,0,0) &= -\frac{s}{2}[\beta(2\eta+1)-2\eta x]\sqrt{1-x^2}\ .
\end{aligned} \tag{3.10}
$$

Notice that, as is well known [14], the transverse cross-section σ_T receives anomalous contributions only from f_1^V, whilst σ_M only from f_3^V. Finally, the longitudinal cross-section σ_L depends on all six anomalous form-factors $f_1^V,\ f_2^V,\ f_3^V$.

Using the expressions given above, we first compute the differential cross-sections and, as a check, we verify that by combining all three we obtain again the results of the previous section. After performing the angular integration in order to obtain the total cross-sections we also check, by setting $f_i^V \to 0$, that our SM result agrees with the polarized cross-sections presented in [17]. After these basic checks of our calculation, we subtract the SM contribution to obtain three new observables

$$
\begin{aligned}
\sigma_5 &\equiv \left[\frac{128\pi s}{g^4\beta}\right](\sigma_{TT}^{exp}-\sigma_{TT}^0)\ , \\
\sigma_6 &\equiv \left[\frac{128\pi s}{g^4\beta}\right](\sigma_{LT}^{exp}-\sigma_{LT}^0)\ , \\
\sigma_7 &\equiv \left[\frac{128\pi s}{g^4\beta}\right](\sigma_{LL}^{exp}-\sigma_{LL}^0)\ .
\end{aligned} \tag{3.11}
$$

Setting for convenience

$$
\mathcal{L} \equiv \ln\left(\frac{1+\beta}{1-\beta}\right)\ , \tag{3.12}
$$

$$
\begin{aligned}
\tau_1 &\equiv -\frac{\eta}{\beta^2}+1+\frac{8\eta}{3}+\beta^2(1+\eta)\ , \\
\tau_2 &\equiv \eta-\beta^2\left(1+\frac{8\eta}{3}\right)-3\beta^4(1+\eta)\ , \\
\tau_3 &\equiv \frac{1}{\beta^2}-\frac{8}{3}-\beta^2\ , \\
\tau_4 &\equiv \frac{4}{\beta^2}+\frac{16}{3}+12\beta^2\ ,
\end{aligned} \tag{3.13}
$$

$$
\begin{array}{ll}
Q^5 \equiv \frac{16\beta^2}{3}\ , & Q^6 \equiv \frac{128\eta\beta^2}{3}\ , \\
Q^7_A \equiv \frac{8}{3}\beta^2(2\eta-1)^2\ , & Q^7_B \equiv \frac{128}{3}\beta^6\eta^4\ , \\
Q^7_C \equiv \frac{128}{3}\beta^2\eta^2\ , & Q^7_D \equiv \frac{64}{3}\beta^4(2\eta-1)\eta^2\ , \\
Q^7_E \equiv \frac{64}{3}\beta^2(2\eta-1)\eta\ , & Q^7_F \equiv \frac{256}{3}\beta^4\eta^3\ ,
\end{array} \tag{3.14}
$$

the polarized observables σ_5, σ_6, and σ_7 are given by

$$\begin{aligned}
\sigma_5 &= L_1^5\ f_1^\gamma + L_8^5\ f_1^Z + Q_{[1][1]}^5\ \left(f_1^\gamma\right)^2 + Q_{[1][8]}^5\ f_1^\gamma f_3^Z + Q_{[8][8]}^5\ \left(f_1^Z\right)^2\ , \\
\sigma_6 &= L_3^6\ f_3^\gamma + L_{10}^6\ f_3^Z + Q_{[3][3]}^6\ \left(f_3^\gamma\right)^2 + Q_{[3][10]}^6\ f_3^\gamma f_3^Z + Q_{[10][10]}^6\ \left(f_3^Z\right)^2\ , \\
\sigma_7 &= \sum_k L_k^7\ c_k + \sum_k \sum_{l\geq k} Q_{[k][l]}^7 c_k c_l\ ,
\end{aligned} \tag{3.15}$$

and the various coefficients appearing in Eq.(3.15) are explicitly given below:

$$\begin{aligned}
L_1^5 &= s_w^2 \left[\tau_3 + \frac{32\beta^2}{3}(s_w^2 + vu) - \frac{1}{2\beta^3\eta^3}\mathcal{L}\right]\ , \\
L_8^5 &= u\left[(v+a)\tau_3 + \frac{32\beta^2}{3}\left(s_w^2 v + ru\right) - \frac{(v+a)}{2\beta^3\eta^3}\mathcal{L}\right]\ , \\
Q_{[1][1]}^5 &= s_w^4\ Q^5\ , \\
Q_{[1][8]}^5 &= 2s_w^2 vu\ Q^5\ , \\
Q_{[8][8]}^5 &= ru^2\ Q^5\ ,
\end{aligned} \tag{3.16}$$

$$\begin{aligned}
L_3^6 &= s_w^2\eta\left[-\tau_4 + \frac{256\beta^2}{3}(s_w^2 + vu) + \frac{2(1+3\beta^2)}{\beta^3\eta^2}\mathcal{L}\right] f_3^\gamma\ , \\
L_{10}^6 &= u\eta\left[-(v+a)\tau_4 + \frac{256\beta^2}{3}(vs_w^2 + ru) + (v+a)\frac{2(1+3\beta^2)}{\beta^3\eta^2}\mathcal{L}\right]\ , \\
Q_{[3][3]}^6 &= s_w^4\ Q^6\ , \\
Q_{[3][10]}^6 &= 2s_w^2 vu\ Q^6\ , \\
Q_{[10][10]}^6 &= ru^2\ Q^6\ ,
\end{aligned} \tag{3.17}$$

$$\begin{aligned}
L_1^7 &= s_w^2(2\eta - 1)\left[\tau_1 - (s_w^2 + vu)\frac{16}{3}\beta^2(2\eta+1) + \frac{1}{2\eta^3\beta^3}\mathcal{L}\right]\ , \\
L_8^7 &= u(2\eta - 1)\left[(v+a)\tau_1 - (s_w^2 v + ru)\frac{16}{3}\beta^2(2\eta+1) + \frac{(v+a)}{2\eta^3\beta^3}\mathcal{L}\right]\ , \\
L_2^7 &= s_w^2\left[4\eta^2\tau_2 + (s_w^2 + vu)\frac{64}{3}\beta^4\eta(2\eta+1) - \frac{2}{\eta\beta}\mathcal{L}\right]\ , \\
L_9^7 &= u\left[(v+a)4\eta^2\tau_2 + (s_w^2 v + ru)\frac{64}{3}\beta^4\eta(2\eta+1) - \frac{2(v+a)}{\eta\beta}\mathcal{L}\right]\ , \\
L_3^7 &= -4s_w^2\left[\tau_1 - (s_w^2 + vu)\frac{16}{3}\beta^2(2\eta+1) + \frac{1}{2\eta^3\beta^3}\mathcal{L}\right]\ , \\
L_{10}^7 &= -4u\left[(v+a)\tau_1 - (s_w^2 v + ru)\frac{16}{3}\beta^2(2\eta+1) + \frac{(v+a)}{2\eta^3\beta^3}\mathcal{L}\right]\ ,
\end{aligned}$$

$$
\begin{aligned}
Q^7_{[1][1]} &= s_w^4 Q^7_A \ , \\
Q^7_{[1][8]} &= 2s_w^2 vu Q^7_A \ , \\
Q^7_{[8][8]} &= ru^2 Q^7_A \ , \\
Q^7_{[2][2]} &= s_w^4 Q^7_B \ , \\
Q^7_{[2][9]} &= 2s_w^2 vu Q^7_B \ , \\
Q^7_{[2][9]} &= ru^2 Q^7_B \ , \\
Q^7_{[3][3]} &= s_w^4 Q^7_C \ , \\
Q^7_{[3][10]} &= 2s_w^2 vu Q^7_C \ , \\
Q^7_{[10][10]} &= ru^2 Q^7_C \ , \\
Q^7_{[1][2]} &= -s_w^4 Q^7_D \ , \\
Q^7_{[1][9]} &= Q^7_{[2][8]} = s_w^2 vu Q^7_D \ , \\
Q^7_{[8][9]} &= ru^2 Q^7_D \ , \\
Q^7_{[1][3]} &= -s_w^4 Q^7_E \ , \\
Q^7_{[1][10]} &= Q^7_{[3][8]} = s_w^2 vu Q^7_E \ , \\
Q^7_{[8][10]} &= ru^2 Q^7_E \ , \\
Q^7_{[2][3]} &= -s_w^4 Q^7_F \ , \\
Q^7_{[2][10]} &= Q^7_{[3][9]} = s_w^2 vu Q^7_F \ , \\
Q^7_{[9][10]} &= ru^2 Q^7_F \ .
\end{aligned}
\tag{3.18}
$$

4 C and P conserving couplings.

In what follows we will focus on the special case where all anomalous couplings satisfy the individual discrete symmetries C, P, and T, i.e. we assume that $f_4^V = f_5^V = f_6^V = f_7^V = 0$. Then, the polarized σ_i for $i = 5, 6, 7$ are given in Eqs.(3.15), while the corresponding unpolarized σ_i for $i = 1, ..., 4$ assume the following form:

$$
\begin{aligned}
\sigma_1 &= L^1_3 f_3^\gamma + L^1_{10} f_3^Z + Q^1_{[3][3]}(f_3^\gamma)^2 + Q^1_{[3][10]} f_3^\gamma f_3^Z + Q^1_{[10][10]}(f_3^Z)^2 \ , \\
\sigma_2 &= L^2_1 f_1^\gamma + L^2_2 f_2^\gamma + L^2_3 f_3^\gamma + L^2_8 f_1^Z + L^2_9 f_2^Z + L^2_{10} f_3^Z + Q^2_{[1][1]}(f_1^\gamma)^2 + Q^2_{[1][2]} f_1^\gamma f_2^\gamma + Q^2_{[1][3]} f_1^\gamma f_3^\gamma \\
&+ Q^2_{[1][8]} f_1^\gamma f_1^Z + Q^2_{[1][9]} f_1^\gamma f_2^Z + Q^2_{[1][10]} f_1^\gamma f_3^Z + Q^2_{[2][2]}(f_2^\gamma)^2 + Q^2_{[2][3]} f_2^\gamma f_3^\gamma + Q^2_{[2][8]} f_2^\gamma f_1^Z \\
&+ Q^2_{[2][9]} f_2^\gamma f_2^Z + Q^2_{[2][10]} f_2^\gamma f_3^Z + Q^2_{[3][3]}(f_3^\gamma)^2 + Q^2_{[3][8]} f_3^\gamma f_1^Z + Q^2_{[3][9]} f_3^\gamma f_2^Z + Q^2_{[3][10]} f_3^\gamma f_3^Z \\
&+ Q^2_{[8][8]}(f_1^Z)^2 + Q^2_{[8][9]} f_1^Z f_2^Z + Q^2_{[8][10]} f_1^Z f_3^Z + Q^2_{[9][9]}(f_2^Z)^2 + Q^2_{[9][10]} f_2^Z f_3^Z + Q^2_{[10][10]}(f_3^Z)^2 \ , \\
\sigma_3 &= L^3_1 f_1^\gamma + L^3_2 f_2^\gamma + L^3_8 f_1^Z + L^3_9 f_2^Z \ , \\
\sigma_4 &= L^4_3 f_3^\gamma + L^4_{10} f_3^Z \ .
\end{aligned}
\tag{4.19}
$$

The following comments are in order:

(i) Notice that the expressions for σ_3 and σ_4 receive no quadratic contributions and are therefore identical to those presented in [12].

(ii) The expressions for σ_1 and σ_4 constitute a system of two equations with two unknowns, f_3^γ and f_3^Z, as was the case in [12], but now the unknown quantities appear quadratically in σ_1. As we will see in a moment, one of the results of this is that the degeneracy between the two systems is improved.

(iii) By measuring the polarized quantities, one would arrive at a system of seven equations for the six unknown form-factors. In fact, the system separates into two sub-systems: One sub-system of three equations $\{\sigma_1, \sigma_4, \sigma_6\}$ with two unknowns $\{f_3^\gamma, f_3^Z\}$, and one sub-system of the remaining four equations involving all six unknowns. One could then attempt a global solution, or use the first sub-system to determine f_3^γ and f_3^Z, and use their values as input in the other. Notice also that the fact that we have three equations for f_3^γ and f_3^Z may reduce or eliminate completely the ambiguities in determining them which originate from the quadratic nature of these equations [18].

Given that $\{\sigma_1, \sigma_4, \sigma_6\}$ constitute an independent sub-system, it is interesting to carry out an elementary study of their correlations, at least within the context of a simple model simulating the statistical behaviour of the anomalous couplings. Such a study is useful since the three observables involved appear to be intrinsically different in nature, at least in as far as their inclusiveness is concerned: σ_1 and σ_4 originate from convolutions of the unpolarized differential cross-section with the corresponding projective polynomials, whereas σ_6 originates from selecting those specific events of the full cross-section that correspond to longitudinally polarized W bosons. We will assume that the two couplings $f_3^\gamma \equiv z_1$ and $f_3^Z \equiv z_2$ obey independently a normal (Gaussian) probability distribution, with mean μ_i and variance δ_i^2, i.e.

$$p_i(z_i, \mu_i, \delta_i^2) = \frac{1}{\delta_i (2\pi)^{\frac{1}{2}}} \exp\left[-\frac{(z_i - \mu_i)^2}{2\delta_i^2} \right] . \tag{4.20}$$

Then, the expectation value $\langle \sigma_i \rangle$ of the observable $\sigma_i,\ i = 1, 4, 6$ is given by

$$\langle \sigma_i \rangle = \prod_{j=1}^{2} \int_{-\infty}^{+\infty} [dz_j] p_j \sigma_i \ , \tag{4.21}$$

the corresponding covariance matrix by

$$V_{ij} = \langle \sigma_i \sigma_j \rangle - \langle \sigma_i \rangle \langle \sigma_j \rangle \ , \tag{4.22}$$

and the correlations r_{ij} by

$$r_{ij} = \frac{V_{ij}}{V_{ii}^{1/2} V_{jj}^{1/2}} \ . \tag{4.23}$$

We will next assume that the Gaussian distribution is peaked around the SM values of the couplings, i.e. $\mu_i = 0$, and will use the elementary results

$$\int_{-\infty}^{+\infty} [dz_i] z_i p_i^{(0)} = 0, \quad \int_{-\infty}^{+\infty} [dz_i] z_i^2 p_i^{(0)} = \delta_i^2, \quad \int_{-\infty}^{+\infty} [dz_i] z_i^4 p_i^{(0)} = \frac{3}{4}\delta_i^4 \ , \tag{4.24}$$

where $p_i^{(0)} \equiv p_i(z_i, 0, \delta_i^2)$.

We next study the correlations r_{ij} in the absence and presence of quadratic corrections. We assume for simplicity that $\delta_1 = \delta_2 = \delta$; actually, the final results do not depend on δ, which cancels out when forming the ratios in Eq.(4.23). The results for some characteristic values of the center-of-mass energy $\sqrt{s}$ are given in Table 1 and Table 2, respectively.

$\sqrt{s}$ (GeV)	180	200	250	300	500
r_{14}	-0.999	-0.998	-0.995	-0.992	-0.988
r_{16}	0.619	0.763	0.894	0.936	0.975
r_{46}	-0.588	-0.719	-0.842	-0.885	-0.931

Table 1 : The correlation coefficients ρ_{ij} as a function of $\sqrt{s}$ in the absence of quadratic corrections.

$\sqrt{s}$ (GeV)	180	200	250	300	500
r_{14}	-0.992	-0.970	-0.850	-0.686	-0.267
r_{16}	0.333	0.482	0.724	0.859	0.986
r_{46}	-0.215	-0.254	-0.253	-0.218	-0.107

Table 2 : The correlation coefficients ρ_{ij} as a function of $\sqrt{s}$ in the presence of quadratic corrections.

We notice that for all values of $\sqrt{s}$ the inclusion of the quadratic terms leads to lower values for the correlations. The elementary analysis presented above can be easily generalized to include all seven observables, thus constructing the full correlation matrix.

(iv) We note that f_1 and f_3 are enhanced at the production threshold ($\beta \to 0$) due to the factors $1/\beta^2$ which survive in their coefficients ($\tau_1,\ \tau_3,\ \tau_4$) in the polarized observables (remember that there is an overall prefactor $\propto \beta$ stemming from phase space). This enhancement cancels in the total cross-section $\sigma = \sigma_T + \sigma_M + \sigma_L$; this known fact furnishes an additional useful check of our calculation. Evidently, the measurement of the polarized cross-sections will be more sensitive to the f_1 and f_3 form-factors at the low-energy LEP2 runs.

(v) Finally one of the unknown photonic deviations f_2^γ can be completely eliminated by resorting to electromagnetic $U(1)$ gauge-invariance; the latter imposes on the deviation form-factors f_1^γ and f_2^γ the relation

$$f_1^\gamma = \eta f_2^\gamma. \tag{4.25}$$

Thus, the number of unknown form-factors appearing in Eq.(4.19) and Eq.(3.15) will be reduced down to five, a fact which should restrict even further any ambiguities stemming from the quadratic nature of the equations.

5 Conclusions

We have obtained explicit expressions of the unpolarized differential cross-section for the production of an on shell W pair keeping the most general structure for the triple gauge boson vertices, namely all fourteen different form-factors which parametrize the deviations from the tree-level SM trilinear gauge vertex. The above explicit result, which contains all linear and quadratic terms in the anomalous couplings, demonstrates that the unpolarized differential cross-section can be expressed in terms of four polynomials in the cosine of the center-of-mass scattering angle, of maximum degree 3, linearly independent and identical to those obtained in the simpler case where

only linear terms of the C,P and T conserving couplings were kept. The corresponding coefficients multiplying these polynomials can be projected out from the differential cross-section; they constitute a set of four observables, whose measurement imposes experimental constraints on the anomalous couplings.

Furthermore, we have augmented the aforementioned set of observables by three additional ones, which correspond to the total cross-sections for obtaining in the final state W bosons with fixed polarization (both transverse, both longitudinal, one transverse and one longitudinal) in the presence of C,P and T conserving anomalous couplings. The experimental value of these observables can be extracted from measurements of the polarization of the final state W bosons.

The proposed observables comprise a set of seven quadratic equations containing fourteen unknowns, which could be simultaneously fitted in order to put global constraints on all anomalous couplings. Alternatively, one could focus exclusively on the subset of anomalous couplings which separately respect the C, P, and T discrete symmetries, thus arriving at an over-constrained system; the latter could be used in order to eliminate possible algebraic ambiguities in the determination of the above couplings, or reduce their correlations. Imposing in addition electromagnetic gauge invariance one can further restrict the number of unknowns in the above system.

It would be interesting to see how this method responds first to simulated and subsequently to real data. A first step towards a full realization of the method has been recently reported, focussing mainly on aspects related to its experimental feasibility [19].

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FIGURE

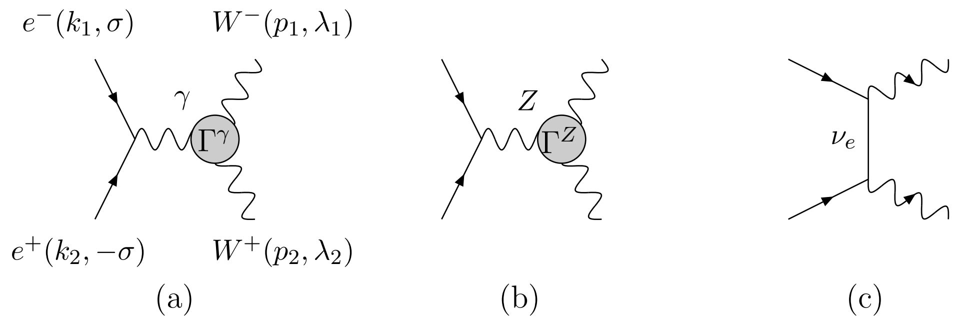

Fig. 1: The process $e^+e^- \to W^+W^-$ at tree level, including anomalous gauge boson couplings.